

INNOVATIONS IN TEACHING OF MATHEMATICS

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Introduction

Though Mathematics being so important subject and occupying a central position since the Ancient period still it has not been the interest of many students. The gaps are found between aspiration and achievement. Mathematics is highly abstract. It is concerned with ideas rather than objects; with the manipulation of symbols rather than the manipulation of object. It is a closely-knit structure in which ideas are interrelated. Mathematical concepts are hierarchical and interconnected, much like a house of cards. Unless lower-level concepts are mastered, higher-level concepts cannot be understood. Students who discover some of the structures of mathematics, are often impressed by its beauty. They note the lack of contradiction, and they see how a new technique can be derived from one that has already been learned.

Teaching of mathematics is not only concerned with the computational know-how of the subject but is also concerned with the selection of the mathematical content and communication leading to its understanding and application. So while teaching mathematics one should use the teaching methods, strategies and pedagogic resources that are much more fruitful in gaining adequate responses from the students then we have ever had in the past.

The teaching and learning of mathematics is a complex activity and many factors determine the success of this activity. The nature and quality of instructional material, the presentation of content, the pedagogic skills of the teacher, the learning environment, the motivation of the students are all important and must be kept in view in any effort to ensure quality in teaching-learning of mathematics.

In this paper the presenter has made an effort to discuss innovations and innovative practices in teaching mathematics, under teaching methods, strategies and pedagogic resources. The process of innovation is generally described as consisting of three essential steps, starting with the conception of an idea, which is then proposed and is finally adopted. Though many ideas have been conceived to bring about change in the teaching of mathematics, it is yet to be proposed and adopted. So, the innovations discussed may not be new in terms of the idea but is new in terms of practice.

Aims of Teaching Mathematics

Education is imparted for achieving certain ends and goals. Various subjects of the school curriculum are different means to achieve these goals. So with each subject some goals are attached which are to be achieved through teaching of that subject. According to Sidhu (1995) the goals of teaching mathematics are as below:

- To develop the mathematical skills like speed, accuracy, neatness, brevity, estimation, etc.
- To develop logical thinking, reasoning power, analytical thinking, critical-thinking.
- To develop power of decision-making.
- To develop the technique of problem solving.
- To recognize the adequacy or inadequacy of given data in relation to any problem.
- To develop scientific attitude i.e. to estimate, find and verify results.
- To develop ability to analyze, to draw inferences and to generalize from the collected data and evidences.
- To develop heuristic attitude and to discover solutions and proofs with the own independent efforts.
- To develop mathematical perspective and outlook for observing the realm of nature and society.

Need for Innovations in Teaching Mathematics

Looking to the aims of teaching mathematics it can be seen that more focus is laid to the higher level of objectives underlying the mathematics subject, like critical thinking, analytical thinking, logical reasoning, decision-making, problem-solving. Such objectives are difficult to be achieved only through verbal and mechanical methods that are usually used in the class of mathematics. The verbal methods of instruction give all importance to speech and texts, to the book and to the teacher. From an historical point of view this method was majorly used until the end of the nineteenth century. In one of these verbal methods teachers are simply satisfied with giving the mathematical rules to pupils and having them memorize it. They justify this method by saying pupils would not understand explanations. Their task is to transmit to their pupils the knowledge which has accumulated over the centuries, to stuff their memory while asking them to work exercises, e.g. The rule of signs and formulas in algebra, students memorize this and remember it! Another verbal method involves explanation. Teachers who use this method assume that the mental structure of the child is same as the adult's. But a developmental stage according to Piaget is a period of years or months during which certain developments take place. Teachers think teaching must imply logic, and logic being linked to language, or at least to verbal thought, verbal teaching is supposed to be sufficient to constitute this logic. This method leads to series of explanations and students at the initial steps of logical explanations trying to understand and grasp but slowly the

gap is created between the explanations transmitted by teacher and received by students which lead to the poor understanding on part of students and they develop a fear of the subject: **Mathsphobia**. The Education Commission (1964-66) points out that "In the teaching of Mathematics emphasis should be more on the understanding of basic principles than on the mechanical teaching of mathematical computations". Commenting on the prevailing situation in schools, it is observed that in the average school today instruction still confirms to a mechanical routine, continues to be dominated by the old besetting evil of verbalism and therefore remains dull and uninspiring.

Innovations in Teaching Mathematics

Innovations in teaching of mathematics can be diversified in terms of Methods, Pedagogic Resources and Mastery Learning Strategy used in teaching-learning process.

1. Mastery Learning Strategy

Teaching Strategy is a generalized plan for a lesson and includes a specific structure to be followed.

B.S. Bloom has developed Mastery Learning Strategy. It is a new instructional strategy that is used for developing mastery learning and objectives of curriculum can be realized. It consists of different steps: division of content into units, formulation of objectives related to each unit, teaching and instruction are organized for realizing objectives of each unit, administering unit test to evaluate the mastery level and diagnose the learning difficulties, remedial instructions are given to remove the difficulties and attain mastery level by every student. This strategy plays an important role for learning of basics and fundamentals e.g. operations in different number systems – Natural numbers, Integers, Rational numbers, Real numbers.

2. Methods

Method is a style of the presentation of content in classroom. The following are the innovative methods that can be used to make teaching-learning process of Mathematics effective.

Inducto-Deductive Method

It is a combination of inductive and deductive method. Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples. In classroom usually the instructions directly start with the abstract concepts and are being taught in a way that does not bring understanding on the part of majority of the students. Formulas, theorems, examples, results are derived, proved and used. But teacher needs to start with specific examples and concrete things and then move to generalizations and abstract things. Then teacher again

needs to show how generalization can be derived and it holds true through specific examples. This method will help students for better understanding, students don't have to cram the things and will have long lasting effect.

Example: Pythagoras Theorem - In a right-angle $\triangle ABC$ right angled at B, $AB^2 + BC^2 = AC^2$ (Considering right angle triangles of different measurement leading to generalization and then establishing it through the theoretical proof).

Analytico-Synthetic Method

It is a combination of Analytic and Synthetic method. Analytic is breaking down and moving from unknown to known and Synthetic is putting together known bits of information and moving from known to unknown. These methods are basically used in proving the results and solving sums. In text-books mostly synthetic method is used, to prove something unknown we start with a certain known thing, but that leaves doubt in mind of students-why we have started with that step and using this particular known thing. So teacher has to use combination in order to explain and relate each step logically.

Example: If $\frac{a}{b} = \frac{c}{d}$ then prove that $d(a-2ab) = b(c-2ad)$.

Synthetic Method	Analytic Method
$\frac{a}{b} = \frac{c}{d}$ $\therefore \frac{a}{b} - 2a = \frac{c}{d} - 2a \text{ (Why??)*}$ $\therefore d(a-2ab) = b(c-2ad)$ <p>*the doubt raised in students mind is being solved with the help of analytic method</p>	$d(a-2ab) = b(c-2ad)$ $\Leftrightarrow \frac{a-2ab}{b} = \frac{c-2ad}{d}$ $\Leftrightarrow \frac{a}{b} - 2a = \frac{c}{d} - 2a$ $\Leftrightarrow \frac{a}{b} = \frac{c}{d}$

Problem-Solving Method

This method aims at presenting the knowledge to be learnt in the form of a problem. It begins with a problematic situation and consists of continuous meaningful well-integrated activity. Choose a problem that uses the knowledge that students already have i.e. you as a teacher should be able to give them the problem and engage them without spending time in going over the things that you think they should know. After students have struggled with the problem to get solution, have them share their solutions. This method will help them in developing divergent thinking.

Example: Put a problem of finding the amount of water in a given container instead of deriving the formula of volume (cylinder filled with water).

Play-Way Method

This method consists of the activities that include a sort of fun or play and give joy to the students. Students don't realize that they are learning but in a way they are gaining knowledge through participating in different activities. This method helps to develop interest in mathematics, motivates students to learn more and reduces the abstract nature of the subject to some extent.

Example: Mathematical games and puzzles.

Laboratory Method

Laboratory method is based on the principles of "learning by doing" and "learning by observation" and proceeding from concrete to abstract. Students do not just listen to the information given but do something practically also. Principles have to be discovered, generalized and established by the students in this method. Students learn through hands on experience. This method leads the student to discover mathematical facts. After discovering something by his own efforts, the student starts taking pride in his achievement, it gives him happiness, mental satisfaction and encourages him towards further achievement.

Example: Making and observing models, paper folding, paper cutting, construction work in geometry.

3. Pedagogic Resources

Pedagogic resources are the resources that a teacher may integrate in a method for the transaction of a particular content and draw upon to advance the students' learning.

Teaching Aids

Teaching aids are the materials used for effective teaching and enhancing the learning of students. It can be anything ready-made or made by the teacher or made by students. Different teaching aids should be used in teaching mathematics like Charts, Manipulatives, Programmed Learning Material (PLM), computers and television.

- **Charts** – It can be used to display formulae, symbols, mathematical and geometrical figures. Charts can be used for making students familiar to the symbols and for memorization of basic formulae. Even it can be used to bring to the students two-dimension geometry and the graphical representation in a better way.
- **Manipulatives** – They are objects or materials that involve mathematics concepts, appealing to several senses, that can be touched and moved around by the students (not demonstrations of materials by the teacher). Each student needs material to manipulate independently. With students actively involved in manipulating materials, interest in mathematics will be aroused. Canny (1984) has shown that mathematics instruction and students' mathematics understanding will be more effective if manipulative materials are used. Models can be used to make things concrete like three dimension figures in geometry.

- **Programmed Learning Material (PLM)** – It is a self-learning material in which learner can proceed at his own pace. It has the characteristics of all sequential steps, learner’s response, self-pacing, immediate feedback, reinforcement and self-evaluation. It is helpful in acquisition of concepts like fractions, number systems, etc. and can be used as a remedy for slow learners for a specific content.
- **Computers and Television** – Computer can be used for multimedia presentation for the concepts that requires visualization and imagination. Computer can also be used for providing Computer Assisted Instruction (CAI), it is similar to PLM i.e. it is a computerized PLM. Television can be used to show some good mathematics education show.

Activities

Activities here include all such work where in students play an active role, has to interact with different resources and generate knowledge. It includes Quiz competition, Projects, Role play, Seminars, Discussion, Mathematics club, Assignment, Field trips, etc.

Name of the Activity	Examples/Situations where Activity can be used
Quiz Competition	Logic, Properties of Numbers, Mathematical Rules and Results
Projects	Contribution by Different Mathematicians
Role Play	Arithmetical concepts like Profit & Loss, Simple & Compound Interest
Seminars	Shortcuts through Vedic Mathematics, Application of Mathematics in other Disciplines
Discussion	Properties of 'Zero', Difference between Rational and Irrational Numbers, Relating Different Concepts in Mathematics
Mathematics Clubs	Application of the concept studied, Preparing Models, Paper Folding (Origami)
Assignment	Self-Study, Extension of Knowledge
Field Trips	Experiencing the Functional use of Mathematics in Bank, Insurance Company

In any curriculum, content and presentation of content are the two most important and inseparable components. It is difficult to say anything definitely about which method and pedagogic resource is going to be most effective for presentation of a particular type of content. Selection of method and pedagogic resource depends on many factors like type of content, objectives to be achieved, level of the students, entry behaviour, availability of resources. Also acceptance of innovative methods and positive attitude of teachers towards it, is an important factor for the selection of method and pedagogic resource. The things included under innovations are existing in books, also there are researches which shows that some innovations are carried out in the classroom and has shown the positive effect on teaching

learning process but their practical usage and implementation in classroom is not seen to the expected level.

Guidelines for a Teacher in Incorporating Innovations in Teaching Mathematics

- For effective transaction of the curriculum and achievement of curricular objectives appropriate method and pedagogic resources should be used in providing learning experiences to the students.
- A number of factors need to be considered while making use of a particular method and pedagogic resource: learners' capabilities, availability of resources, entry behavior, school environment, objectives to be achieved, the nature of content and the teacher's own preparation and mastery.
- Decide on and plan in advance the innovative idea that the teacher would be incorporating to transact a particular concept so that loss of instructional time is prevented or minimized.
- The immediate environment of the learner both natural and human should be used when and where possible for making learning concrete and meaningful.
- Involve the students in the process of learning by taking them beyond the process of listening to that of thinking, reasoning and doing.
- In order to promote self-study skills use of library and resource center needs to be encouraged.
- Receiving regular feedback for teaching and learning should be an inbuilt component of teaching-learning process. Continuous and comprehensive evaluation has to be ensured as it plays an important role for the required modification in teaching-learning process.
- Mathematics-teachers' organizations at different levels should be formed where sharing of ideas and experiences, developing resources in a collaborative manner and the mechanisms that enable teachers to carry out innovations is being discussed. Mathematics-teachers' organizations can be instrumental in establishing a climate of confidence in carrying out innovations and a positive attitude to new approaches in teaching mathematics.
- Properly instruct and guide the students for carrying out different activities and precautionary measures should be taken so that students are not misguided.
- Study mathematical journals and modern books of professional interest. Any facilities of in-service training should be availed of for improving teaching of mathematics.

The teacher can always ask himself two questions: 1. 'Is there some new way in which I can present this material in order to make it more meaningful and more interesting?' 2. 'What activities, demonstrations, teaching aids, etc. would enrich the classroom presentation and direct attention of students to the important elements?' Once the teacher discovers innovative ways to

arouse interest and enthusiasm in the class, he will be able to use these ideas again the following year, since those will be new and fascinating to a different class. But teacher should keep in mind that as time passes, the world undergoes a change, the environment surrounding students changes and their needs also changes, so one has to continuously go on modifying and discovering new ways of teaching which proves him a better teacher.



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Summary. The aim of this contribution is to present some innovative aspects of teaching mathematics at secondary level in order to face two key issues an innovative teaching: why and how. To Paolo Abrantes, in memoriam.

1. Introduction. Our aim in this presentation is to face the interesting questions of "why?" and "how?" we can approach innovative teaching of mathematics at secondary level.
2. Innovations in the curriculum. We need to teach well what people need to learn. But learning needs are related to people's future. This claim justifies that curriculum must be sensitive to changes. First of all we may consider the innovation of goals and contexts, i.e., new concepts and topics to be learnt as well as new competencies to be achieved (Niss, 2001). In contemporary education, mathematics education is the practice of teaching and learning mathematics, along with the associated scholarly research. Researchers in mathematics education are primarily concerned with the tools, methods and approaches that facilitate practice or the study of practice; however, mathematics education research, known on the continent of Europe as the didactics or pedagogy of mathematics, has developed into an extensive field of study, with its concepts, theories, methods. The famous mathematician John von Neumann once said, "In mathematics, you don't understand things; you just get used to them." However, I don't agree with him. I believe that if we develop projects that are hands-on learning experience which should be included in the schools curriculum. Sirkku is Head of Mathematics at Helsinki University Viikki Teacher Training School, Finland. She has been at the forefront of the enhancement of mathematics education in Finland over the past two decades and is a mentor teacher for student teachers at the University Practice School. sirkku.myllyntausta@helsinki.fi.

Yvonne Beston. We are grateful to all the contributors and their colleagues whose work has enabled the Centre for Innovation in Mathematics Teaching (CIMT) to compile this publication. In particular, we are grateful to Jodie Hunter who undertook the initial editing of many of the contributions, and to our funders, CfBT Education Trust, for their continued support of initiatives vital to the enhancement of education, both in the UK and worldwide. You can't teach in a vacuum. Collaborate with other teachers to improve your math instruction skills. Start by discussing the goal for the math lesson, what it will look like, and plan as a team to be most effective. "Together, think through the tasks and possible student responses you might encounter," says Andrews. Reflect on what did and didn't work to improve your practice. What do you feel are the most important strategies in teaching mathematics? We'd love to hear. Share your ideas in our WeAreTeachers HELPLINE group on Facebook.