Cosmic Black-body Radiation in Plasma Universe

Dirk K. Callebaut
Physics Dept., Campus Drie Eiken
University of Antwerp
B-2610 Antwerp, BELGIUM
e-mail: Dirk.Callebaut@ua.ac.be

ABSTRACT

In the previous papers[1-5] it was shown that Planck's radiation law has to be generalized in a homogeneous and isotropic plasma in thermal equilibrium. A new parameter $a = ¦\omega_p / k_B T$ occurs, where ω_p is the angular plasma frequency and ¦, k_B and T have their customary meaning. The effect on the relic cosmic black-body radiation after the separation between radiation and matter (at about 4000 K) seems to be measurable in the (near) future.

1. INTRODUCTION

Landau-Lifshitz[6], when starting the section on black-body radiation, state: "The most important application of Bose statistics is to electromagnetic radiation". Further they state: "It must be borne in mind that the presence of some matter, however little, is in general necessary to enable thermal equilibrium to be set up in the radiation, since one may assume that there is no interaction between the photons. (Apart from the completely negligible interaction due to the possible existence of virtual electron – positron pairs.)" However, when plasma is present, it interacts with the radiation. The electric permittivity and the magnetic permeability may change with respect to their values in vacuum. However, here we are interested in another aspect of the interaction: the dispersion relation for transversal waves (photons) reads

$$\omega^2 = k^2 c^2 + \omega_p^2 \tag{1}$$

Here ω is the angular frequency of the photons, k their wavenumber, c the velocity of light and ω_p is the angular plasma frequency:

$$\omega_p = \sum_i \frac{q_i^2 n_i}{\varepsilon\, m_i} \tag{2}$$

with q_i the charge, n_i the number density and m_i the mass of the $i-$th species of particles. The similarity and differences with the dispersion relation for longitudinal waves may be recalled:

$$\omega^2 = k^2 v_s^2 + \omega_p^2 \tag{3}$$

where v_s is the sound velocity. Note that equation (3) is slightly adapted when e.g. statistical refinements or nonlinear effects are considered, while equation (1) is nearly insensitive to such refinements, what is important for the ensuing considerations. Equations (1) and (3) show that ω has a lower bound: putting $k = 0$ leaves us with $\omega^2 = \omega_p^2$. The region $(0, \omega_p)$ is forbidden. This makes clear that the black-body radiation has to be changed when plasma is present: "plasma-body" radiation generalizes black-body radiation.

In section 2 we deal with some aspects of plasma-body radiation. In section 3 we consider applications, essentially the "cosmic black-body radiation" (CBBR) or rather CPBR.

2. PLASMA – BODY RADIATION

The theory has been developed essentially in references 1, 2 and 3, using the theory of statistical physics [6]. Here we give some results and some comments only.

2.1. Photon mass

In view of equation (1) the energy $\in_\gamma = \hbar\omega$ of a photon may be written as

$$\in_\gamma = c\left(p_\gamma^2 + m_\gamma^2 c^2\right)^{1/2} \tag{4}$$

where $p_\gamma = k\hbar = \hbar\left(\sqrt{\omega^2 - \omega_p^2}\right)\Big/c$, $m_\gamma = \hbar\omega_p / c^2$, $\hbar = h/2\pi$ and h is a Planck's constant. The form of this energy is identical with the relativistic expression for the total energy of material particles. This allows us to interpret m_γ as the rest mass of a photon in a plasma. In contrast to the rest mass of material particles this rest mass is variable: it depends on ω_p and hence on the density of plasma (mainly) as well as on q_i and m_i. In the theory of relativity we learned that the mass of a body depends on its speed. However, the rest mass did not yet vary, while here it does. The unexpected surprise of this effect is the greater as we are dealing with photons, which have no rest mass in vacuum. The following analogy may be helpful: a sponge in a liquid changes its mass too. The analogy is not perfect, but it should be clear that an electromagnetic wave acts on a plasma, e.g. it causes (small) currents, and those in turn act on the wave, e.g. by endowing it with an induced mass. Another, probably better, analogy is the polaron.

2.2. Total radiation energy

For the total kinetic energy density of the photon gas we obtained [1,2]:

$$u_k = \frac{T^4 a^3}{4\pi^2(\hbar c)^3}\sum \frac{e^{ja}}{j}\left[(1+ja)K_1(ja) + (3-ja)K_3(ja)\right] \tag{5}$$

where

$$a = \frac{\hbar\omega_p}{k_B T} \tag{6}$$

and $K_n(x)$ is the Bessel function of the second kind (imaginary argument) and of n th order.

For $a \ll 1$ (low densities and/or high temperatures) we obtain for the total energy density:

$$u = \frac{T^4}{\pi^2(\hbar c)^3}\left[6\zeta(4) + 4\zeta(3)a\right] \tag{7}$$

where $\zeta(a)$ is the Riemann zeta function, with $\zeta(3) = 1.202\ldots$ and $\zeta(4) = \pi^4/90$. Introducing the constant of Stefan $\sigma = \pi^2/60\hbar^3 c^2$, we obtain

$$u = \frac{4\sigma}{c}T^4\left(1 + \frac{72.12a}{\pi^4}\right). \tag{8}$$

Since a varies inversely with temperature, we have, besides the customary term proportional to T^4 a term proportional to T^3. Moreover the latter term is proportional to ω_p, hence to $n^{1/2}$. The additional term is relevant at high densities and low temperatures. When $a \gg 1$ another approximation is appropriate[2].

It is at once clear that in the plasma – body radiation the laws of Stefan – Boltzmann and Wien have to be adapted.

2.3. Quantitative effect

Qualitatively speaking the change is remarkable. However, the quantitative effect of the generalization to plasma – body radiation is usually very limited, except in the centre of stars, in particular of White Dwarfs. Two reasons contribute to the small quantitative effect:

a) The parameter a is usually small.
b) The radiation distribution of Planck approaches zero like ω^3 when ω approaches zero. Hence the contribution which is deleted by the cut-off frequency ω_p is very small. This however is overcompensated because the volume element in the frequency space for $\omega > \omega_P$ is larger than for $\omega < \omega_P$. (Hence the energy density increases with increasing a.) However the (over)compensation remains very small as long as $a << 1$, i.e. when $\hbar\omega_p << k_B T$. In fact this parameter measures the relative importance of ω versus T, or of ω_p versus $\omega_{\max}$, the frequency where the maximum of the radiation law occurs. (This $\omega_{\max}$ increases slowly when ω_p increases). However, once $a \approx 1$ the contribution becomes substantial. For White Dwarfs one may have $a \approx 200$, causing a gigantic change.

2.4. Other quantities

We derived similar expressions[1-5] for the free energy, the pressure, the specific heat, etc.

3. APPLICATIONS

3.1 Various application

Several applications were considered before[1-5]: the effect in the centre of the Sun, or in general a star. E.g. in the Sun the radiation energy and pressure are increased by some 20% with respect to the black-body radiation. However, it should be noted that the radiation pressure in the centre of the Sun is of the order of 10^{-3} of the kinetic pressure. Nevertheless, as the radiation is the basic means of radiation transport inside the Sun (or star) below the convective zone this affects seriously the heat transport and hence the conventional idea of stellar structure and internal temperature.

For White Dwarfs the radiation pressure may be two orders of magnitude higher than with the black-body radiation. However, again this pressure is quasi negligible as compared to the other pressures there. However, again the heat transport may be affected seriously.

Another case is constituted by pulsar atmospheres [7,8] which may have densities of $10^{32}/\mathrm{m}^3$, however with temperatures around 10^{10} K, still resulting in a small a. However here we are mainly interested in a cosmic application.

3.2 Cosmic application

3.2.1. Present

Thanks to COBE the cosmic temperature, i.e. the temperature of the cosmic black-body radiation (CBBR) at present has a temperature of 2.726 K (absolute measurement). Let us assume, to fix the ideas, that the density amounts to 10 electrons/m^3 in the intergalactic space. Then we obtain $\omega_p = 200$ Hz and $a \approx 5 \cdot 10^{-10}$. The correction to the energy density as given by equation (8) is then:

$3.5 \cdot 10^{-10}$. However, the (relative) measurements were able to indicate variations in the range of 10^{-6} only (!)

3.2.2. Relics from the separation of radiation and matter

This is the most interesting case, as COBE in fact observes rather the relic radiation of the time when the temperature was about 4000 K.

During the expansion of the universe we may assume that the density varied inversely with the third power of R, the radius of the universe (or whatever characteristic length). Thus ω_p varies like $R^{-3/2}$. On the other hand the temperature varies like R^{-1}. Hence a varies like $R^{-1/2}$ or like $T^{-1/2}$. 4000 K is about 1500 times the present temperature of CBBR, increasing a with a factor 40. We obtain $a = 2 \cdot 10^{-8}$ and the correction term in equation (8) yields $1.5 \cdot 10^{-8}$. The measurements by COBE were not able to detect such variations, let alone absolute measurements. However, there is a hope for the future.

3.2.3. Still older relics

If we go back in the cosmic history, e.g. when the temperature of the universe was $3 \cdot 10^{10}$ K we obtain $a = 5 \cdot 10^{-5}$ and the correction to the energy density is $3.5 \cdot 10^{-5}$. This is still small. Moreover observable effects may have been washed out. Remounting the history to $T = 3 \cdot 10^{18}$ K yields $a = 0.5$, yielding a substantial correction. Going back in time further increases the contribution to a multiple of the value given by the black-body radiation.

4. CONCLUSION

Although the plasma – body radiation involves qualitatively a profound change from the black – body radiation, its quantitative effects are not yet measurable in the cosmic radiation. There is hope that the plasma correction may be observed in a (near) future. The situation is in strong contrast with the one of White Dwarfs where the radiation energy density may be increased by two orders of magnitude.

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A blackbody is a theoretical or model body which absorbs all radiation falling on it, reflecting or transmitting none. It is a hypothetical object which is a â€œperfectâ€ absorber and a â€œperfectâ€ emitter of radiation over all wavelengths.Â In 1965, the cosmic microwave background radiation (CMBR) was discovered by Penzias and Wilson, who later won the Nobel Prize for their work. The radiation spectrum was measured by the COBE satellite and found to be a remarkable fit to a blackbody curve with a temperature of 2.725 K and is interpreted as evidence that the universe has been expanding and cooling for about 13.7 billion years. Since the radiation emitted by a blackbody is isotropic (the same in all directions), it holds that the intensity (power per unit area) of radiation is simply. I=cu4 I = \frac{cu}4. I=4cu .Â The cosmic microwave background (CMB) originates from hot plasma in the early universe. Suppose we treat the source of the CMB as a blackbody. If the universe expands at a constant rate such that its volume. V V. V is given by. At early times the Cosmic Background Radiation photons had enough energy to produce particle-antiparticle pairs and a simple thermal equilibrium would have essentially the same number of each species of particle and corresponding antiparticle as photons in the very early universe.Â The early universe is dominated by the cosmic background radiation (CBR) photons which interact strongly with the electrons (electrically coupÂ led to the protons and helium nuclei) to make what is called the photon-baryon plasma.Â Large-angular-scale anisotropies in the 3 K primordial black-body radiation were detected and mapped with a sensitivity of 2 x 10-4 K and an angular resolution of about 10 deg. The CMB radiation stands for Cosmic Microwave Background Radiation - It is basically the afterglow of the universe. The temperature of the CMB Radiation is...Â The fact that the universe is expanding is one of the most profound discoveries of the 20th century. This means that the universe was much smaller, denser, and hotter in the distant past. At such high density and temperature, the universe was in the plasma state, and matter and radiation were in thermal equilibrium.Â The CMB radiation has a temperature of 2.7 K and its spectrum is a thermal black body curve. Since birth 13.7 billion years ago, the universe is continuously expanding and cooling. After 400,000 years from the start of the universe, it cooled down to a temperature of 3000 degrees Celsius.