Teaching Special Relativity: Minkowski trumps Einstein

EXTENDED ABSTRACT

Teaching Special Relativity: Minkowski trumps Einstein

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Students find physics difficult---I am thinking of first-year undergraduate university physics majors. I found it difficult myself, and it took me almost 40 years of teaching physics to fully understand the reasons for the perceived "difficulty." *Why* do students who find mathematics *easy* to understand, find physics *difficult* to understand?

There are two parts to the answer. First, and most importantly, by "understand" students mean "fit into my existing correct understanding of what the world is." Since the students' existing understanding of what the world is, is, in fact, quite *incorrect*; and since what is correct is almost impossible for the average person to believe; there is a gigantic barrier to understanding physics. (The barrier is, however, entirely psychological.)

Secondly, the physics that they are being taught is not *correct* physics; it is engineering approximations only. So, not only does what they are being taught not make sense in terms of the students' (incorrect) worldview, it does not even make sense in terms of a correct quantum-mechanical worldview.

It is a wonder that anyone at all *ever* sticks to physics, and grasps it.

Correct physics is of course quantum mechanics. Quantum mechanics has effortlessly survived intense attacks by the most brilliant of physicists; quantum mechanics will at minimum be with us for my lifetime and yours; it is the basis of all of our current thinking, and it provides our current *conception* of the universe, encapsulated in the headline "Measurements Are the Only Reality, Say Quantum Tests" (Glanz 1995).

Non-relativistic quantum mechanics leads to Maxwell's equations, which are Lorentz invariant (Dyson 1990). After quantum mechanics itself, Minkowski's union of space and time into spacetime is the greatest advance that has ever occurred in our understanding of the nature of the universe (that is, of the observations).

How grotesquely badly we *teach* special relativity encapsulates the practical problem of teaching physics to the freshman physics major. I have never found a single freshman physics textbook that teaches Minkowski spacetime; I have never found a single text on General Relativity that mentions "Einstein's two postulates."

Every physics freshman is taught ... well, let me quote an example. In the fall of 2007 I will, for the second time in my career, teach introductory physics for physical science majors at the Johns Hopkins University. One text that has recently been used for that course is "University Physics," by R. L. Reese. On page 1155 we read "The entire special theory stems from only two postulates . .... **Postulate 1:** The speed of light in a vacuum has the same numerical value *c* when measured in any inertial reference frame, independent of the motion of the source and/or observer." ... **Postulate 2:** The fundamental laws of physics must be the same in all inertial reference frames."

The reader is invited to recoil, not only at the bizarre re-numbering of the infamous two postulates, but of course at the use of the postulates at all.

There is no doubt that, historically, Albert Einstein, in 1905, *did* introduce *two* postulates (and also, that it is he who discovered special relativity). But the second of these postulates (the one concerning the constancy of *c*, just in case Reese has confused you!) did not survive the year. In September of 1905 Einstein published a development from relativity---the discovery of the implication that $E = m\,c^2$, and in this new paper he mentions a single postulate only. But the paper contains a sweet footnote: "The principle of the constancy of the velocity of light is of course contained in Maxwell's equations." How I love that "of course!" Einstein was human!

I do not know if it is true, but I recall being told that during the Middle Ages undergraduates learned to multiply and divide using Roman numerals, while the exotic Arabic numerals were reserved for the more advanced students.

That is exactly what we do today in teaching special relativity. Antique postulates that are not of anything but historical interest to genuine physicists are presented to students as "Special Relativity." Some books do better than others in warning students how seemingly impossible the second postulate is; but all have the students working out true but unintuitive consequences (e.g. relativity of simultaneity) using thought experiments with of course the second postulate producing the bizarre result.

A small number of texts (Ohanian, Knight, a few others) at least follow Einstein's second paper in having but a single postulate; but none do what needs to be done, which is to drop Einstein and adopt Minkowski.

I feel that the time has come to relegate the "two postulates" to the dustbin of history, and to teach special relativity to undergraduates (or indeed, to middle school students) the Minkowski way.

I believe it is around grade nine that students are taught the Pythagorean Theorem. It is taught as mathematics, and no hint is given that "the keys to the kingdom" are in hand. That happens, of course, because the teacher does *not know* special relativity.

All that is additionally required is the recognition that one can, if one wishes, measure distances using imaginary numbers. And, that the same is true for *time*! Students react well to the suggestion that time is the fourth dimension. And then I say, "I can visit Rome, but I cannot visit Julius Caesar. So we need some *distinction* between space and time. Suggestions, please?"

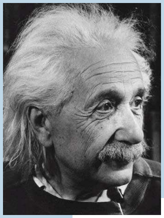
Almost always, after some Socratic prodding, I do get "use imaginary numbers for one, and real numbers for the other?" I reward the student by announcing "you have just discovered Einstein's theory of special relativity!"

The notion that the Minkowski metric describes spacetime is entirely plausible to the students. It is not counter-intuitive, in any way, at least on its face. I announce to the students that experiment is the test!

Once students agree that the Minkowski metric *might* describe the world, it is of course extremely easy to deduce that *if* this should be *our* world, then there must be a limiting velocity. Everything else then follows with ease, and the students emerge comfortable with special relativity as a marvelous insight into the mathematical structure of the observations---the observations that we naively interpret as a universe.

REFERENCES:

Glanz, J. 1995, Science 270, 1439
Dyson, F. J. 1990, Amer. J. Phys. 58, 209



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The Case of the Disappearing Postulate!

On the Electrodynamics of Moving Bodies
by A. Einstein, **1905 June**

We will raise this ... Principle of Relativity to the status of a postulate, and also introduce another postulate ...that light is always propagated in empty space with a definite velocity *c* which is independent of the state of motion of the emitting body. These two postulates ...

Does the Inertia of a Body Depend upon its Energy-Content?
by A. Einstein, **1905 September**

... I based that investigation on the Maxwell-Hertz equations ... and the principle that:- *The laws by which the states of physical systems alter are independent of the alternative, to which of two systems of coordinates, in uniform motion of parallel translation relatively to each other, these alterations of state are referred (principle of relativity.)*
With these principles* as my basis, I deduce ...

*The principle of the constancy of the velocity of light is of course contained in Maxwell's equations.

$$d\tau^2 = dt^2 - dx^2$$

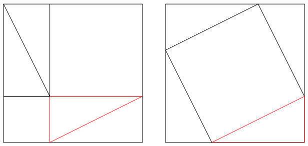
$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

c is NOT the velocity of light - it is merely a poor choice of units

SPACE AND TIME
by H. Minkowski, 1908 September 21

The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

PROOF WITHOUT WORDS

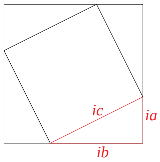


PROOF WITH ALGEBRA

$$\left[(i\,a + i\,b)^2\right] = \left[(i\,c)^2 + 4\left(\frac{1}{2}(i\,a)(i\,b)\right)\right]$$

$$-a^2 - 2ab - b^2 = -c^2 - 2ab$$

$$-a^2 - b^2 = -c^2$$



IMAGINARY MULTI-DIMENSIONAL GEOMETRY

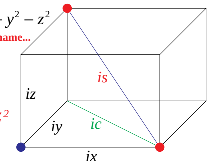
$$-s^2 = -t^2 - x^2 - y^2 - z^2$$

A rose by any other name...

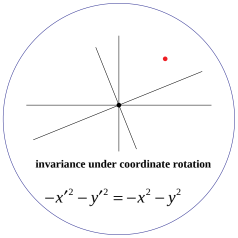
$$-c^2 = -x^2 \; -y^2$$

$$-s^2 = -c^2 \; -z^2$$

$$-s^2 = -x^2 -y^2 -z^2$$

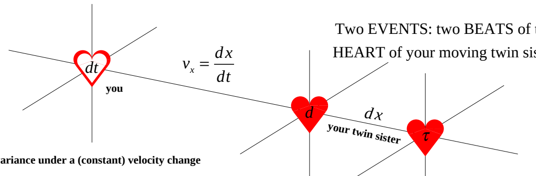


WE LEARN, FROM MINKOWSKI, THAT THE SECRET OF THE UNIVERSE IS: JUST HIGH SCHOOL MATH !



invariance under coordinate rotation

$$-x'^2 - y'^2 = -x^2 - y^2$$



Two EVENTS: two BEATS of the HEART of your moving twin sister

invariance under a (constant) velocity change

m is the invariant mass of your sister

$$m^2 c^2 d\tau^2 = m^2 c^2 dt^2 - m^2 dx^2$$

$$m^2 c^2 = (mc\gamma)^2 - m^2 \left(\frac{dx}{d\tau}\right)^2$$

$$m^2 c^2 = \frac{E^2}{c^2} - m^2 u^2 \qquad u_x \equiv \frac{dx}{d\tau}$$

$$E^2 = (pc)^2 + (m\,c^2)^2 \qquad p_x \equiv m u_x$$

$$mc\gamma = mc\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$= \frac{mc^2}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right)$$

$$= \frac{1}{c}\left(mc^2 + \frac{1}{2} m v^2 + \cdots\right)$$

$$\equiv \frac{1}{c}\left(mc^2 + K\right) \equiv \frac{1}{c}(E)$$

$$c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

<------------------ sis ------------------> <--------------- you --------------->

$$c^2 d\tau^2 - 0^2 - 0^2 - 0^2 = c^2 dt^2 - dx^2 - 0^2 - 0^2$$

$$c^2 d\tau^2 = c^2 dt^2 - dx^2$$

$$c^2 \left(\frac{d\tau}{dt}\right)^2 = c^2 - \left(\frac{dx}{dt}\right)^2$$

$$dt = \frac{d\tau}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma\, d\tau$$

Time dilation: when your twin sister returns, she is younger than you.
Note that *v*<*c*, or a real number would equal an imaginary number!

if your sister is not moving, her kinetic energy is zero, and her ENERGY is:

$$E = mc^2$$

... the most famous equation in the history of the world!

2008 February 02. On behalf of the Program Committee of the Third International Conference on the Nature and Ontology of Spacetime to be held on June 13-15, 2008 in Montreal I would like to inform you that your proposed paper has been accepted for poster presentation. Like in the other conferences the Program Committee cannot send referees reports, but in the biennial spacetime conferences all effort has been made to make the reviewing process as objective as possible - each extended abstract has been independently reviewed by six or in case of discrepancies by seven reviewers.

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In special relativity (as in general relativity), space and time are fused together into a single four-dimensional entity known as spacetime. The words of Hermann Minkowski - a german mathematician and also Einstein's professor at Zurich polytechnic - delivered at the 80th Assembly of German Natural Scientists and Physicians (21 September 1908), and by which he introduced spacetime to the world, are now famous: "The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. How grotesquely badly we teach special relativity encapsulates the practical problem of teaching physics to the freshman physics major. I have never found a single freshman physics textbook that teaches Minkowski spacetime; I have never found a single text on General Relativity that mentions "Einsteinâ€™s two postulates." Every physics freshman is taught well, let me quote an example. In the fall of 2007 I will, for the second time in my career, teach introductory physics for physical science majors at the Johns Hopkins University. How grotesquely badly we teach special relativity encapsulates the practical problem of teaching physics to the freshman physics major. I have never found a single freshman physics textbook that teaches Minkowski spacetime; I have never found a single text on General Relativity that mentions "Einstein's two postulates." Every physics freshman is taught well, let me quote an example. In the fall of 2007 I will, for the second time in my career, teach introductory physics for physical science majors at the Johns Hopkins University. Einsteinâ€™s special relativity and particularly Minkowskiâ€™s interpretation of the relativity principle according to which ob-servers in relative motion have dierent times and spaces (whereas an abso-lute space implies a single space). However, Minkowski had not been concerned about such an apparent contradiction at all. He provided rigorous criteria for inertial and acceler-ated motion56 â€" a free particle, which moves by inertia, is a straight timelike worldline in Minkowski spacetime, whereas the timelike worldline of an ac-celerating particle is clearly dierent â€" it is curved (i.e. deformed)